**E**stimation **A**nd **C**onfidence **I**ntervals

**Background**

In quality control processes, especially when dealing with high-value items, destructive sampling is a necessary but costly method to ensure product quality. The test to determine whether an item meets the quality standards destroys the item, leading to the requirement of small sample sizes due to cost constraints.

**Scenario**

A manufacturer of print-heads for personal computers is interested in estimating the mean durability of their print-heads in terms of the number of characters printed before failure. To assess this, the manufacturer conducts a study on a small sample of print-heads due to the destructive nature of the testing process.

**Data**

A total of 15 print-heads were randomly selected and tested until failure. The durability of each print-head (in millions of characters) was recorded as follows:

1.13, 1.55, 1.43, 0.92, 1.25, 1.36, 1.32, 0.85, 1.07, 1.48, 1.20, 1.33, 1.18, 1.22, 1.29

**Assignment Tasks**

**a. Build 99% Confidence Interval Using Sample Standard Deviation**

Assuming the sample is representative of the population, construct a 99% confidence interval for the mean number of characters printed before the print-head fails using the sample standard deviation. Explain the steps you take and the rationale behind using the t-distribution for this task.

**b. Build 99% Confidence Interval Using Known Population Standard Deviation**

If it were known that the population standard deviation is 0.2 million characters, construct a 99% confidence interval for the mean number of characters printed before failure.

**Answer :-**

**(a): Building a 99% Confidence Interval Using Sample Standard Deviation**

1. **Understanding the Data and Requirements:**

* We have a sample of 15 print-heads with their durability (in millions of characters) recorded.
* We need to construct a 99% confidence interval for the mean durability using the sample standard deviation.

1. **Steps to Calculate the Confidence Interval:**

* **Calculate the Sample Mean:** The sample mean is the average durability of the print-heads in your sample.
* **Calculate the Sample Standard Deviation:** This measures the variability or spread of the durability values in your sample.
* **Determine the t-value:** For a 99% confidence level and 14 degrees of freedom (since you have 15 samples), you look up the t-value from the t-distribution table. The t-distribution is used because the sample size is small, and the population standard deviation is unknown.
* **Calculate the Margin of Error:** The margin of error is calculated by multiplying the t-value by the sample standard deviation divided by the square root of the sample size. This gives you the range of values that account for the variability in your sample.
* **Construct the Confidence Interval:** The confidence interval is the sample mean plus or minus the margin of error. This interval provides a range within which the true mean durability of the print-heads is expected to fall with 99% confidence.

**Summary for Part (a):**

Using the sample standard deviation, the 99% confidence interval is approximately between 1.090 and 1.387 million characters. This means you can be 99% confident that the true mean durability of the print-heads lies within this range. The t-distribution is used because the sample size is small and the population standard deviation is unknown.

**(b): Building a 99% Confidence Interval Using Known Population Standard Deviation**

**1. Understanding the Data and Requirements:**

* + It is given that the population standard deviation is 0.2 million characters.
  + You need to construct a 99% confidence interval for the mean durability using this known population standard deviation.

1. **Steps to Calculate the Confidence Interval:**
   * Calculate the Sample Mean: Use the same sample mean calculated in part (a).
   * Determine the z-value: For a 99% confidence level, you look up the z-value from the z-distribution table. The z-distribution is used when the population standard deviation is known.
   * Calculate the Margin of Error: The margin of error is calculated by multiplying the z-value by the population standard deviation divided by the square root of the sample size. This accounts for the known variability in the population.
   * Construct the Confidence Interval: The confidence interval is the sample mean plus or minus the margin of error. This interval provides a range within which the true mean durability of the print-heads is expected to fall with 99% confidence.

**Summary for Part (b):**

Using the known population standard deviation, the 99% confidence interval is approximately between 1.106 and 1.372 million characters. This means you can be 99% confident that the true mean durability of the print-heads lies within this range. The z-distribution is used because the population standard deviation is known.

**Overall Summary**

In both tasks, the confidence interval provides a range within which you can be 99% confident that the true mean durability of the print-heads falls. The method differs based on whether the population standard deviation is known or not. For small sample sizes and unknown population standard deviation, the t-distribution is used. When the population standard deviation is known, the z-distribution is used.